

**13.13** Given the following functions  $F(s)$ , find  $f(t)$ .

$$(a) \quad F(s) = \frac{s^2 + 5s + 4}{(s + 2)(s + 4)(s + 6)}$$

$$(b) \quad F(s) = \frac{(s + 3)(s + 6)}{s(s^2 + 8s + 12)}$$

**SOLUTION:**

$$a) \quad F(s) = \frac{(s+4)(s+1)}{(s+2)(s+4)(s+6)} = \frac{(s+1)}{(s+2)(s+6)} = \frac{k_1}{s+2} + \frac{k_2}{s+6}$$

$$k_1 = \frac{-2+1}{-2+6} = -1/4 \quad k_2 = \frac{-6+1}{-6+2} = 5/4$$

$$F(s) = \frac{1}{4} \left[ \frac{s}{s+6} - \frac{1}{s+2} \right] \Rightarrow \boxed{f(t) = \frac{1}{4} [se^{-6t} - e^{-2t}] u(t)}$$

$$b) \quad F(s) = \frac{(s+3)(s+6)}{s(s+6)(s+2)} = \frac{s+3}{s(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+2}$$

$$k_1 = \frac{3}{2} \quad k_2 = \frac{-2+3}{-2} = -1/2$$

$$F(s) = \frac{1}{2} \left[ \frac{3}{s} - \frac{1}{s+2} \right] \Rightarrow \boxed{f(t) = \frac{1}{2} [3 - e^{-2t}] u(t)}$$